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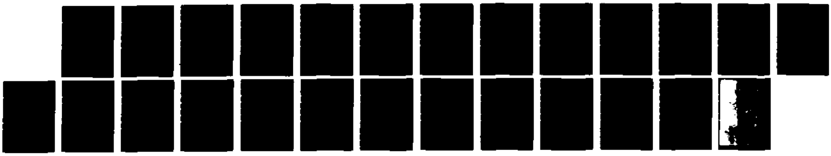

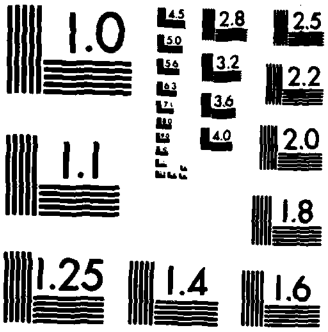

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NAVAL UNDERWATER SYSTEMS CENTER
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Technical Memorandum

INPUT DEFLECTION REQUIREMENTS FOR QUANTIZERS FOLLOWED BY GREATEST-OF DEVICE AND INTEGRATOR

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ABSTRACT

Derivations and programs are presented for evaluation of the required input deflection, for specified output deflection, of a system composed of quantizers followed by a Greatest-Of device and an integrator. The important parameters are: L, the number of levels in each quantizer (along with their breakpoints and step heights); M, the number of statistically independent samples summed in the integrator; N, the number of statistically independent input channels; and d_0, the system output deflection. Several numerical examples are presented. Extension of some earlier results for a system without quantizers (Ref. 1) is also made, and additional results given.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

Practical realizations of desired systems often incorporate approximations to ideal devices, for the sake of expense and equipment complexity. In particular, quantization is frequently employed, since it facilitates data processing. In an earlier study (Ref. 1), the required input signal-to-noise ratio for a Greatest-Of device followed by an integrator was determined; here, we wish to extend the analysis to a system which also has quantization prior to the Greatest-Of device. Surprisingly, it will turn out that the analysis of this more-complicated data processor is much simpler and quicker (via computer aid) than that of the earlier system without quantization. It is advantageous for the reader to be familiar with the assumptions, notation, and results of Ref. 1 before reading this memorandum.

SYSTEM DEFINITION AND ASSUMPTIONS

The system of interest is depicted in figure 1. The input channels $x_1, \ldots, x_N$ are assumed statistically independent and contain either (a) no signal in any channel, or (b) signal in one channel only. (A method for handling statistically-dependent inputs, by defining an effective number of statistically-independent channels, is described in appendix A. Thus the present results are applicable to a wider class of inputs than presumed here, under proper interpretation of the value of N.) The probability density functions of a single channel random variable x, under the two hypothesis (a) and (b), are denoted by $p_0(x)$ and $p_1(x)$, respectively, for the noise-only channels and the signal-bearing channel. The noise-only channels are identically-distributed; however, the analysis could be extended to the case of different distributions on each channel.

The Greatest-Of device is described mathematically by

$$y = \max\{f(x_1), \ldots, f(x_N)\}. \tag{1}$$

Now we shall limit consideration to non-linear no-memory devices f that are monotonically non-decreasing; that is

$$f(a) \leq f(b) \quad \text{if} \quad a < b. \tag{2}$$

Then (1) can be written in the equivalent form

$$y = f\left(\max\{x_1, \ldots, x_N\}\right). \tag{3}$$

This implies that the system in figure 2 is equivalent to that of figure 1, when (2) is satisfied. In practice, figure 1 might be preferred for a quantizer f, because the Greatest-Of device need only handle a discrete set of inputs, rather than the continuous inputs in figure 2. The reason for pointing out (3) is that the processor in figure 2 is more convenient to analyze than figure 1; however, the system of figure 1 is capable of analysis for a general non-monotonic quantizing nonlinearity f, as will become apparent by the methods to follow.

The output of the integrator in figures 1 and 2 is assumed to consist of a sum of M statistically-independent samples. (For a discrete sum of dependent samples, an effective number of independent samples can be defined in a manner identical to that in Appendix A; see Ref. 2, Appendix B. Alternatively, a continuous integration can also be modelled as an equivalent discrete summation of an effective number of independent samples, by a slight modification of Appendix A; see for example, Ref. 3, Appendix A. Thus, the present results are applicable to a wider class of integrators than presumed here, under proper interpretation of the value of M.)

DEFINITION OF OUTPUT DEFLECTION

Since the integrator output is

$$z = \sum_{m=1}^{M} y_m , \tag{4}$$

we have for its mean and variance, respectively,

$$\mu_z = M \mu_y , \quad \sigma_z^2 = M \sigma_y^2 , \tag{5}$$

using the independence of samples $\{y_m\}$. (Actually, we need only require $\{y_m\}$ be uncorrelated.)

We define a system output deflection (prior to the threshold comparison) as the physically-meaningful quantity

$$d_o = \frac{\mu_{z1} - \mu_{z0}}{\sigma_{z0}}, \tag{6}$$

where subscripts 0 and 1 denote, respectively, the signal-absent and signal-present cases. In an earlier study (Ref. 1), we presumed that output z was Gaussian (as it would reasonably be, for large M). The way this assumption appeared in the results of the earlier analysis was in the setting of the value of d_o in (6), for prescribed false alarm and detection probabilities; see Ref. 1, eqs. (6)-(16). Here, we will not assume z is Gaussian; rather, we will simply require deflection d_o to take values in the neighborhood*, 3-5, knowing that relatively good performance, in terms of false alarm and detection probabilities, is then attainable through appropriate choice of threshold T in figure 1. The actual numerical values selected for d_o for plotting purposes will, however, correspond exactly to those used earlier*, for comparison purposes. The purpose of this discussion is to point out that the results actually have applicability to a wider class of outputs, z, than presumed in the original work (Ref. 1), under proper interpretation of d_o as a physically-useful (although statistically incomplete) measure of performance.

Substituting (5) in (6), we have

$$d_o = \sqrt{M}\ \frac{\mu_{y1} - \mu_{y0}}{\sigma_{y0}}. \tag{7}$$

SPECIALIZATION TO QUANTIZERS

In order to evaluate (7), we need to evaluate the moments (see figure 2)

$$\overline{y^\nu} = \overline{f^\nu(w)} = \int dw\, q(w)\, f^\nu(w), \tag{8}$$

*For example, $-\Phi^{-1}(P_F) = 3.090$ and 4.753 for $P_F = 10^{-3}$ and 10^{-6}, respectively; see Ref. 1, eqs. (16) and (10).

where q(w) is the probability density function of w. (We will impose subscripts 0 and 1 when necessary.) For general monotonic nonlinearities f, this is a complicated expression. However, when f is a quantizer, (8) takes a particularly simple form. Consider the general monotonic quantizer characterization in figure 3. Without loss of generality, $L \geq 1$; otherwise, the quantizer output is independent of its input. Also $h_\ell < h_{\ell+1}$ for $0 \leq \ell \leq L - 1$, and $b_\ell < b_{\ell+1}$ for $1 \leq \ell \leq L-1 \, (L \geq 2)$.

Then (8) becomes simply*

$$\overline{y^\nu} = h_0^\nu \int_{-\infty}^{b_1} dx\, q(x) + \sum_{\ell=1}^{L-1} h_\ell^\nu \int_{b_\ell}^{b_{\ell+1}} dx\, q(x) + h_L^\nu \int_{b_L}^{\infty} dx\, q(x)$$

$$= h_0^\nu Q(b_1) + \sum_{\ell=1}^{L-1} h_\ell^\nu \left[Q(b_{\ell+1}) - Q(b_\ell)\right] + h_L^\nu \left[1 - Q(b_L)\right] \tag{9}$$

$$= h_L^\nu - \sum_{\ell=1}^{L} Q(b_\ell)\left[h_\ell^\nu - h_{\ell-1}^\nu\right],$$

where Q(w) is the cumulative distribution function of w. In particular, the quantities needed in (7) are (re-establishing subscripts)

$$\mu_{y0} = h_L - \sum_{\ell=1}^{L} Q_0(b_\ell)\left[h_\ell - h_{\ell-1}\right],$$

$$\mu_{y1} = h_L - \sum_{\ell=1}^{L} Q_1(b_\ell)\left[h_\ell - h_{\ell-1}\right],$$

$$\overline{y_0^2} = h_L^2 - \sum_{\ell=1}^{L} Q_0(b_\ell)\left[h_\ell^2 - h_{\ell-1}^2\right], \tag{10}$$

$$\sigma_{y0} = \left(\overline{y_0^2} - \mu_{y0}^2\right)^{1/2}.$$

The cumulative distribution functions of w under the signal-absent and signal-present cases, respectively, are, using the statistical independence of $\{x_1, \ldots, x_N\}$ (see figure 2),

$$Q_0(w) = P_0^N(w),$$

$$Q_1(w) = P_0^{N-1}(w) P_1(w), \tag{11}$$

*By contrast, the results for no non-linear device, i.e. $f(x) = x$, are much more complicated; see Ref. 1, eqs. (17), (18), and (30) (upper line).

where $P_0(x)$ and $P_1(x)$ are the cumulative distribution functions of x under the signal-absent and signal-present alternatives, respectively; see Definitions subsection above.

SPECIALIZATION TO GAUSSIAN INPUTS

This specialization to Gaussian inputs is made solely for the purpose of evaluating (10) and (11) for a particular example. It is not fundamental, and can be replaced by another more-appropriate example if desired. For Gaussian inputs, we have

$$P_0(x) = \Phi\left(\frac{x-m_0}{\sigma}\right),$$
$$P_1(x) = \Phi\left(\frac{x-m_1}{\sigma}\right), \qquad (12)$$

where m_0, m_1 are the means, and σ is the standard deviation (assumed identical), for the two signal hypotheses. It is convenient to define an input deflection (analogous to output deflection (6)) as

$$d_i = \frac{m_1-m_0}{\sigma} . \qquad (13)$$

(This quantity was denoted by r in Ref. 1). Employing (11) and (12) in (10), we can now evaluate

$$Q_0(b_\ell) = \Phi^N\left(\frac{b_\ell-m_0}{\sigma}\right),$$
$$Q_1(b_\ell) = \Phi^{N-1}\left(\frac{b_\ell-m_0}{\sigma}\right)\Phi\left(\frac{b_\ell-m_1}{\sigma}\right). \qquad (14)$$

The quantizer break-points $\{b_\ell\}$ will be specified in terms of input parameters m_0 and σ (see figure 1); i.e., let

$$b_\ell = m_0 + \sigma v_\ell \ , \ \text{or} \ \frac{b_\ell - m_0}{\sigma} = v_\ell \ , \ 1 \le \ell \le L, \qquad (15)$$

where v_ℓ is a "normalized" breakpoint. Then (14) becomes

$$Q_0(b_\ell) = \Phi^N(v_\ell),$$
$$Q_1(b_\ell) = \Phi^{N-1}(v_\ell)\,\Phi(v_\ell - d_i), \qquad (16)$$

where we also utilized (13). The $\{v_\ell\}$ in (15) can be negative or positive.

SUMMARY OF ANALYTICAL RESULTS FOR QUANTIZERS PRESENT

Output deflection d_0 is given by (7). The quantities needed in (7) are given by (10). And the latter quantities are given by (16). The necessary input parameters are L, $\{h_\ell\}_0^L$, $\{v_\ell\}_1^L$, M, N, d_i. A program for the evaluation of d_0 is given in appendix B. We will set d_0 equal to 3.090 or 4.753, which as noted earlier, correspond to $P_F = 10^{-3}$, $P_D = .5$ or $P_F = 10^{-6}$, $P_D = .5$ respectively, and solve for the required value of input deflection d_i, for specified quantizer, number of input channels, and number of independent samples in the integrator.

The quantizer example to be investigated is an eight-level (3 bit) device as shown in figure 4. That is, all the steps are of equal spacing and height, and extend from the noise-only mean m_0 to $m_0 + 2\sigma$. Thus from figure 3 and (15),

$$v_\ell = \frac{\ell - 1}{3}, \quad 1 \le \ell \le L = 7. \qquad (17)$$

We will characterizer this spacing by $\Delta v = 1/3$.

RESULTS

The input deflection, d_i, required for M = 16 is presented in figure 5, as a function of N, the number of independent input channels. Similar results for M = 32 and 64 are presented in figures 6 and 7.

Since the quantizer in figure 4 only spans the range (m_0, $m_0 + 2\sigma$), it is anticipated that better performance might be attained if the quantizer covered a wider range. In figure 8, a comparison of quantizers (for M = 32) which cover the range (m_0, $m_0 + 6\,\Delta v$) is made, for Δv = 1/3, .5, and .7. It is

observed that best performance (lowest d_i) is realized when $\Delta v = .5$; that is, the quantizers then cover the range $(m_0, m_0 + 3\sigma)$. Results for other cases are easily available by means of the program in appendix B. In particular, some of the $\{v_k\}$ could be negative if desired.

EXTENSION OF EARLIER RESULTS

In Ref. 1, analytical and graphical results were presented for the system without quantizers. In particular, in Ref. 1, eqs. (21) and (31), the quantities

$$a_N = N \int dx \, x \, \phi(x) \, \Phi^{N-1}(x),$$
$$b_N = N \int dx \, x^2 \phi(x) \, \Phi^{N-1}(x), \tag{18}$$

were found necessary. By use of some results in Ref. 4, closed form expressions for some of these quantities have been found, which augment eqs. (22) and (32) in Ref. 1. They are listed in Table 1 below.

N	a_N	b_N
1	0	1
2	$1/\sqrt{\pi}$	1
3	$1.5/\sqrt{\pi}$	$1+\frac{\sqrt{3}}{2\pi}$
4	$\frac{3}{\sqrt{\pi}}\left(1-\frac{2}{\pi}\arctan\frac{1}{\sqrt{2}}\right)$	$1+\frac{\sqrt{3}}{\pi}$
5	$\frac{5}{\sqrt{\pi}}\left(1-\frac{3}{\pi}\arctan\frac{1}{\sqrt{2}}\right)$	?

Table 1. Values of a_N and b_N

The graphical results in Ref. 1 were for very large values of M. Here we present some additional results* for M = 16, 32, 64, 128, in figures 9-10, respectively. The labelling on these figures corresponds to the deflection criteria cited earlier in this memorandum, rather than that in Ref. 1. These results also allow for comparison with the quantizer results given in figures 5-8 above.

*A program for the system without quantizers is given in Appendix C.

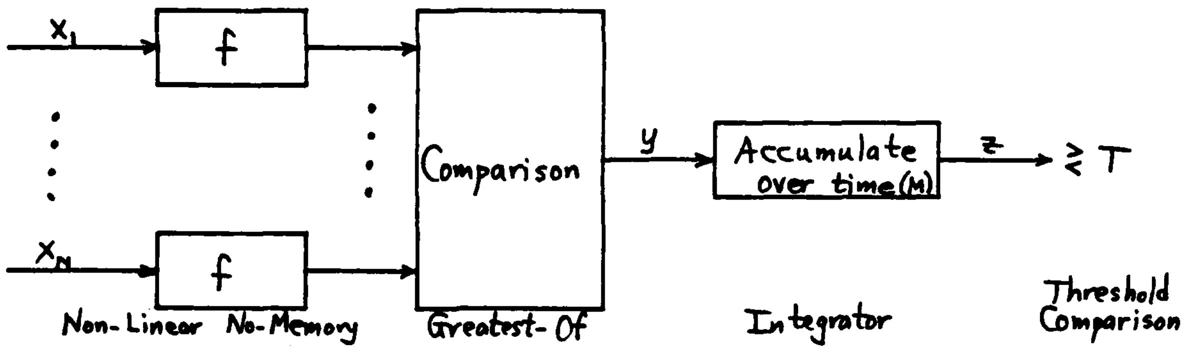

Figure 1. Generic Block Diagram

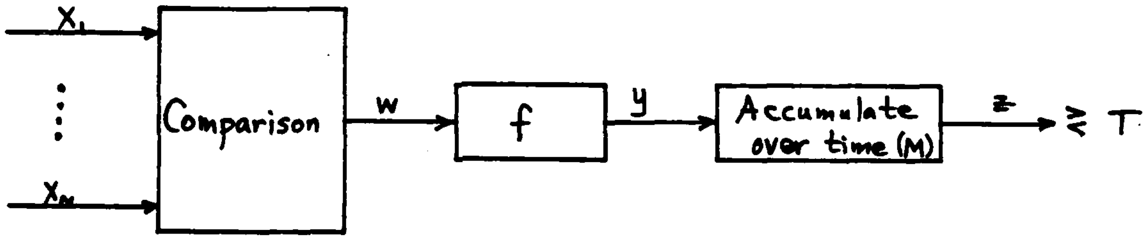

Figure 2. Equivalent Processor For Monotonic Non-Linearity

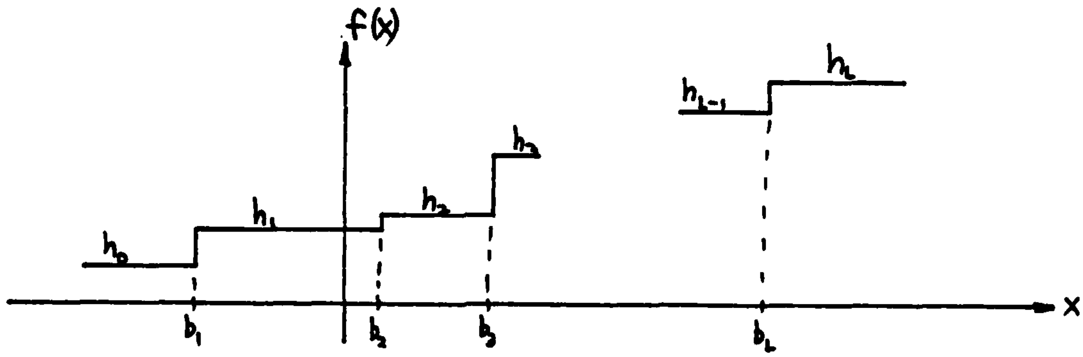

Figure 3. Quantizer Characterization

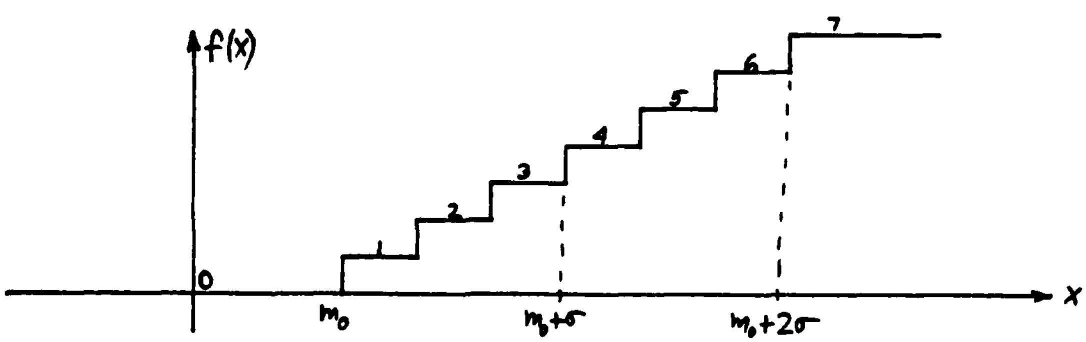

Figure 4. Quantizer Example

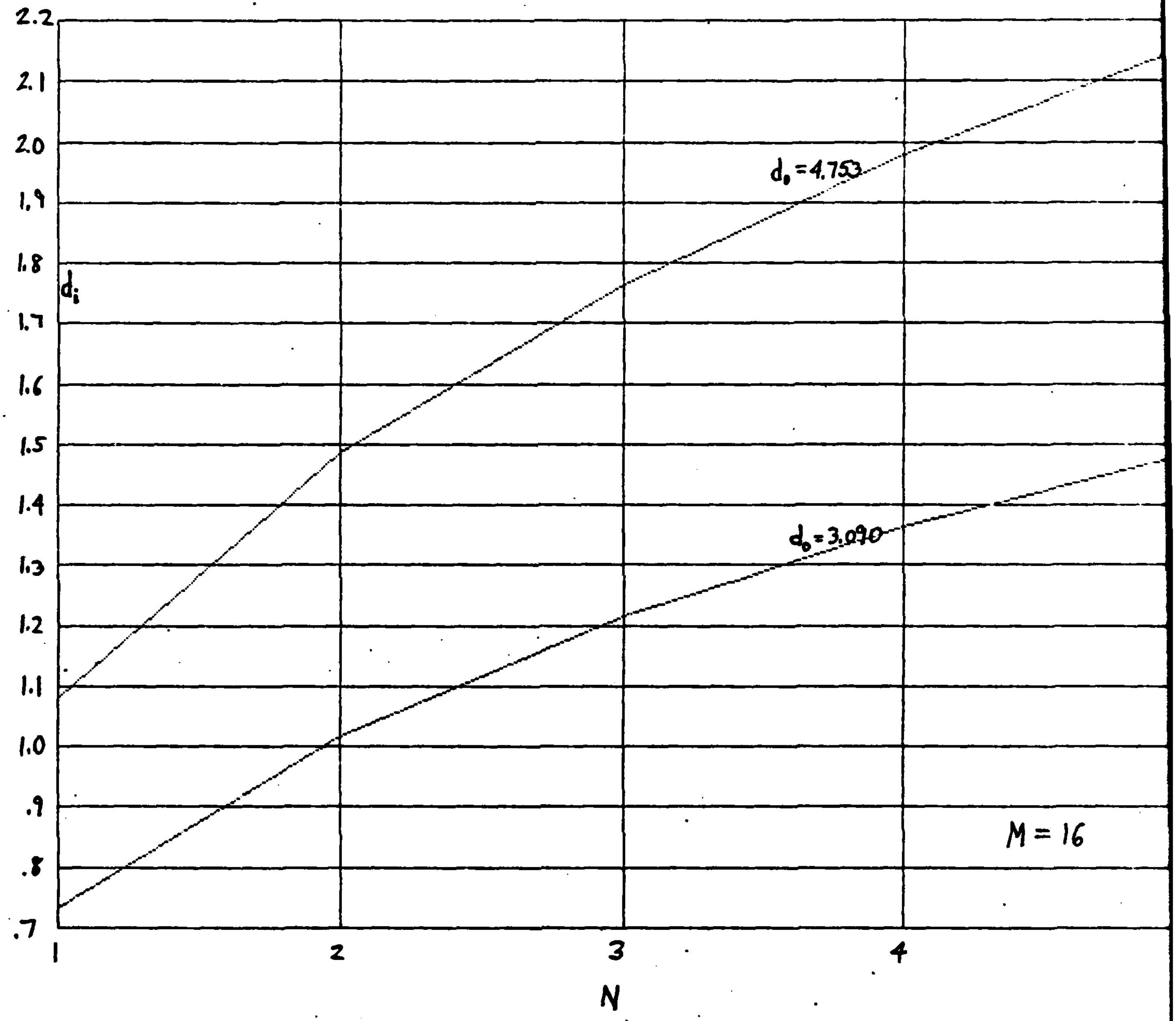

Figure 5. Input Deflection Requirements for M = 16

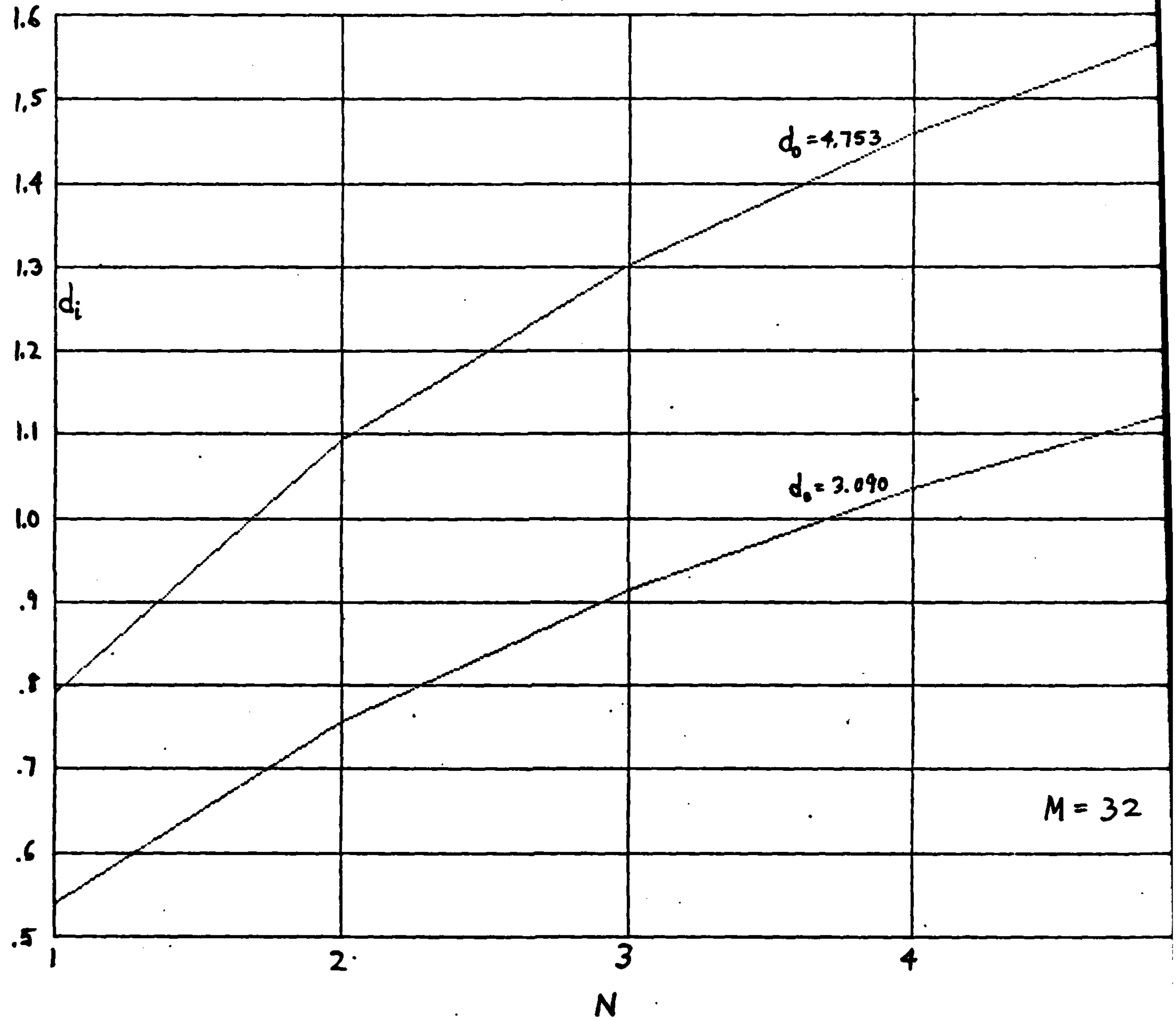

Figure 6. Input Deflection Requirements for M = 32

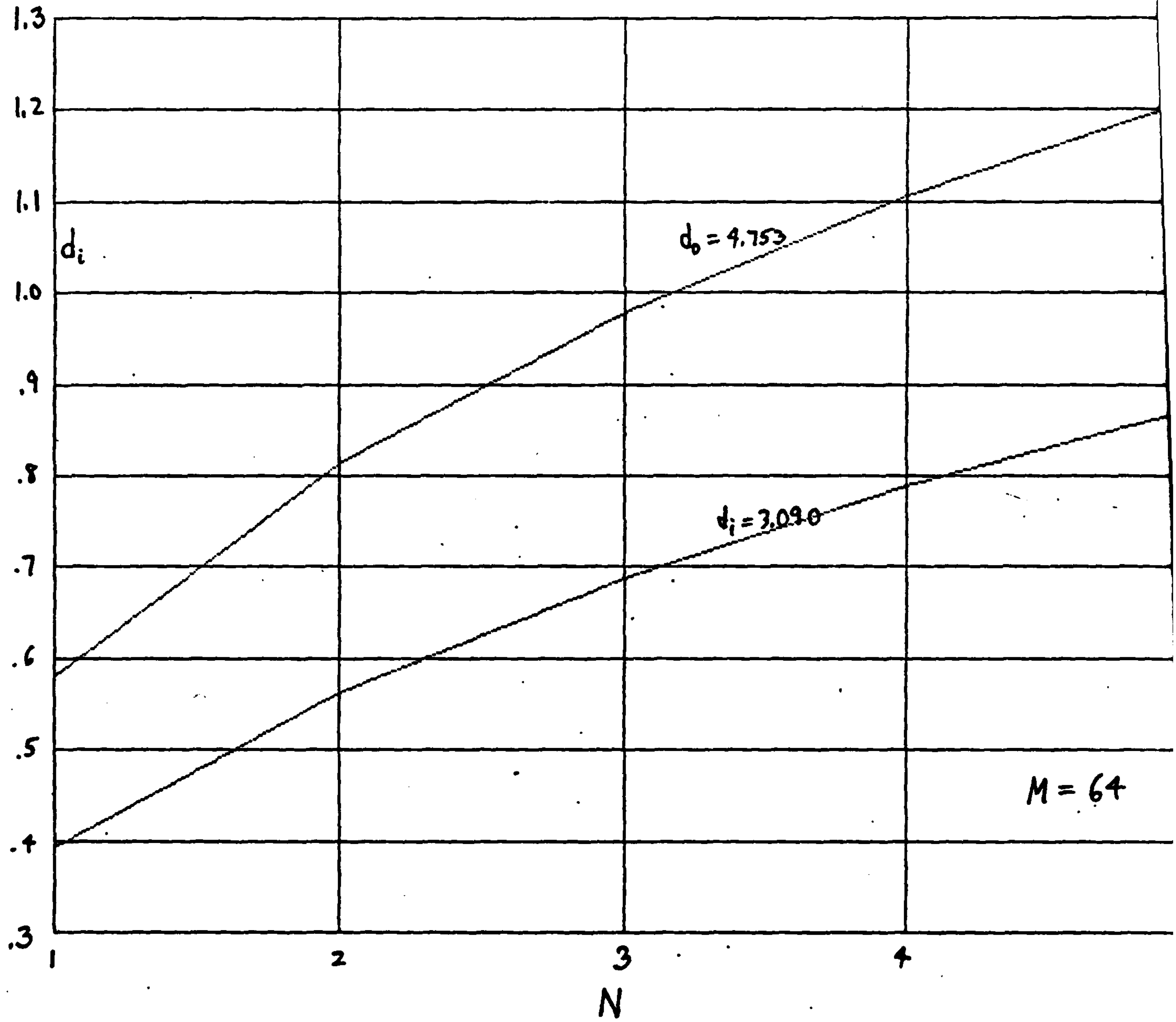

Figure 7. Input Deflection Requirements for M = 64

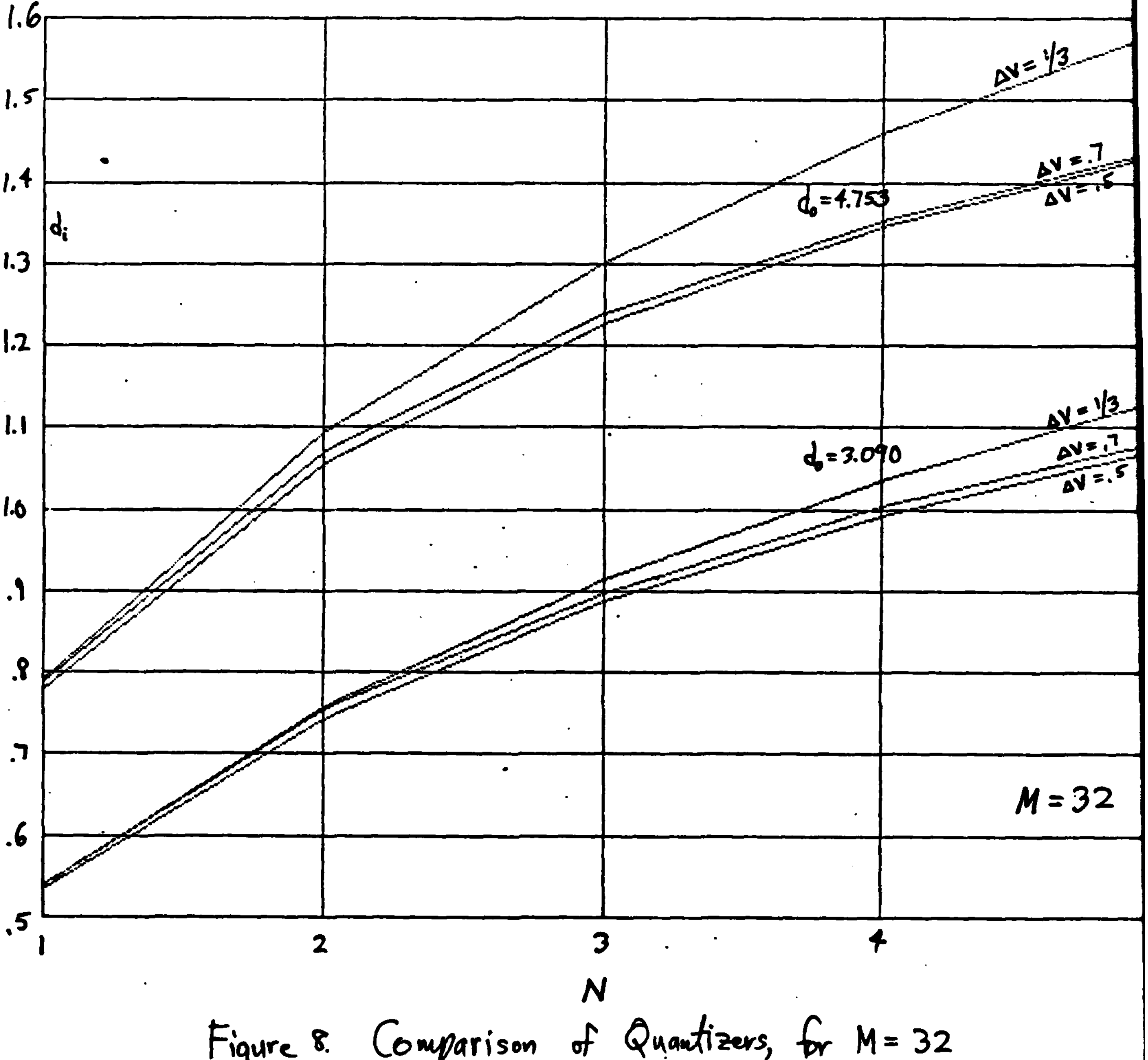

Figure 8. Comparison of Quantizers, for M = 32

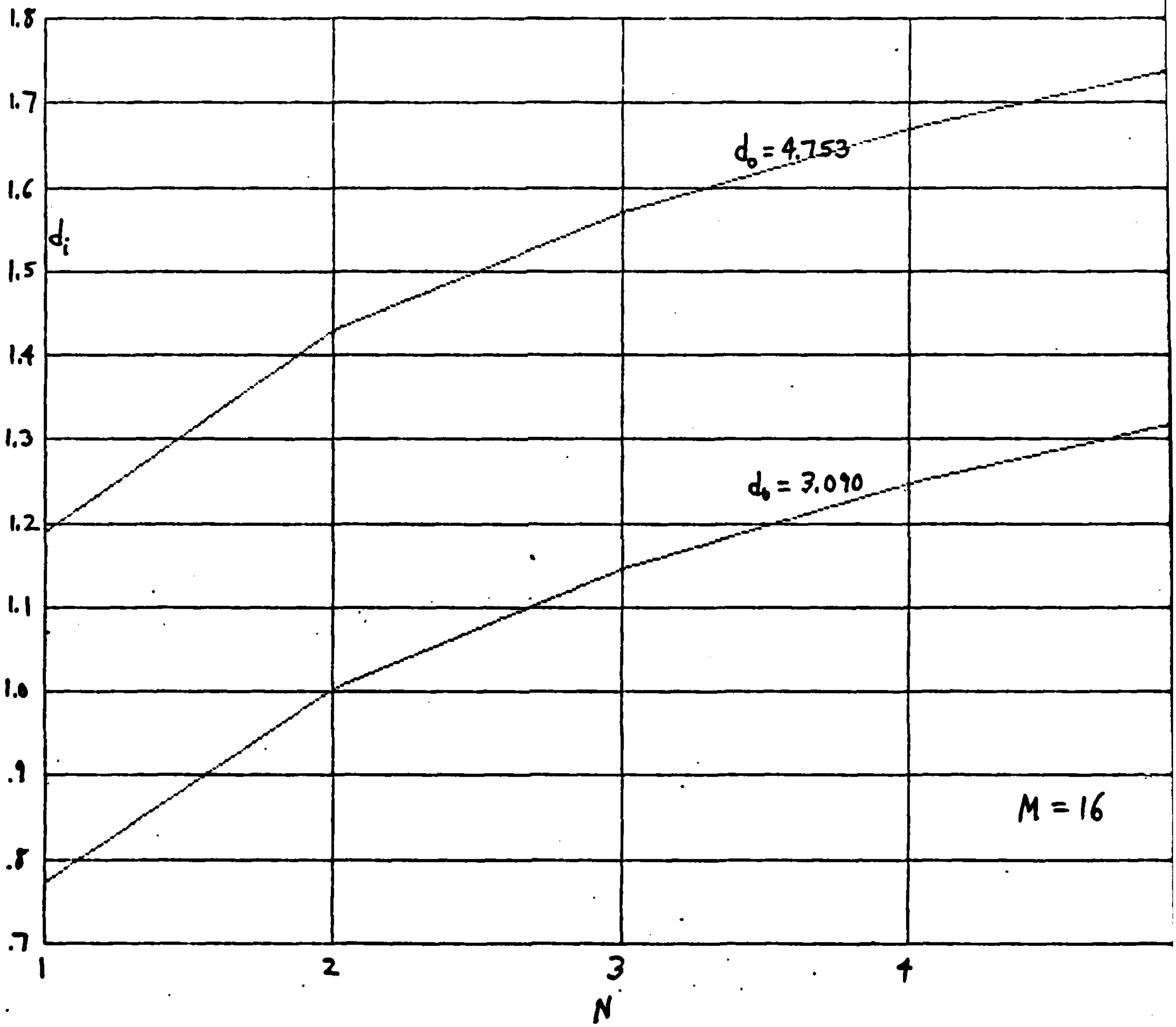

Figure 9. Input Deflection Requirements for M = 16, No Quantizers

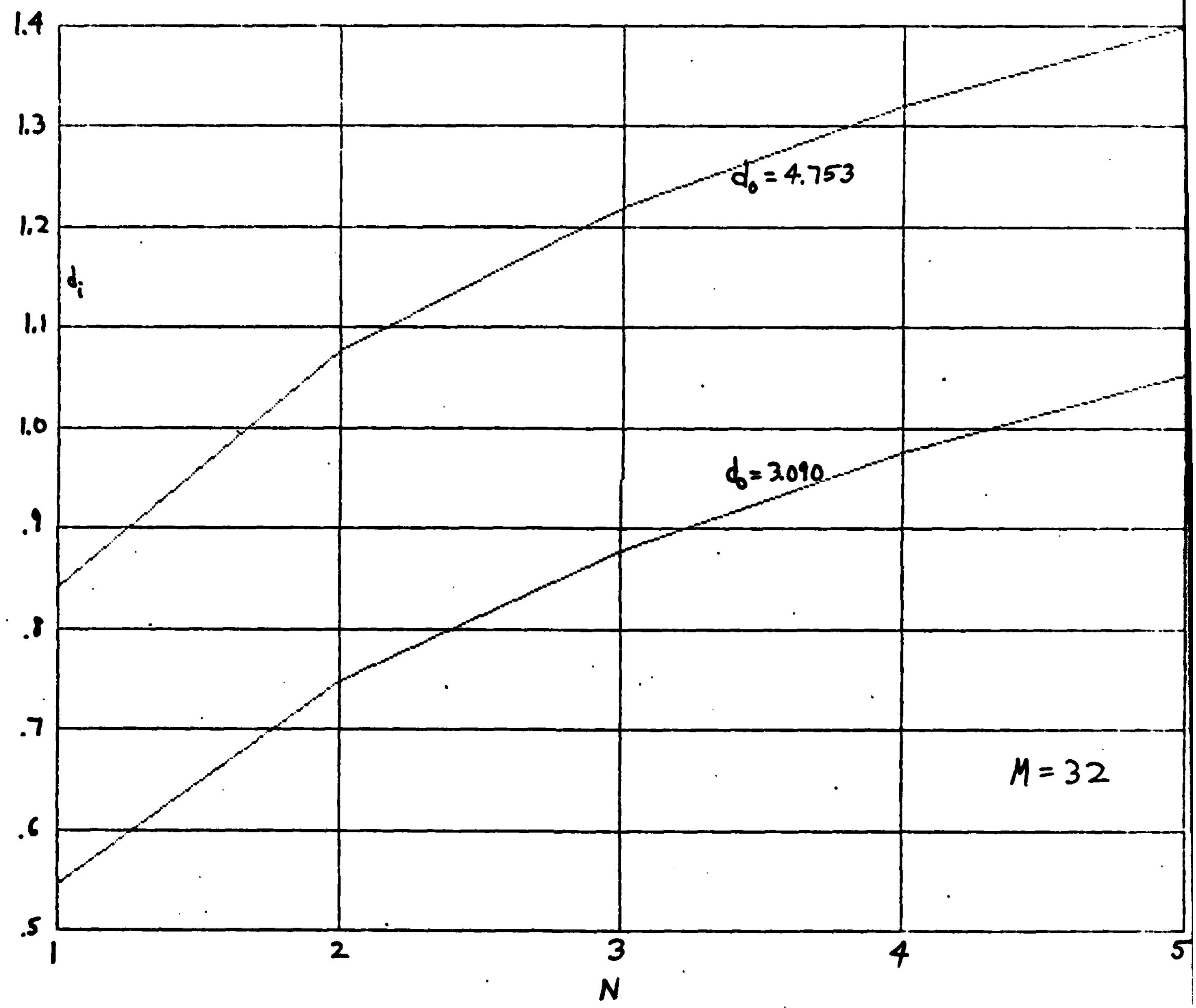

Figure 10. Input Deflection Requirements for M=32, No Quantizers

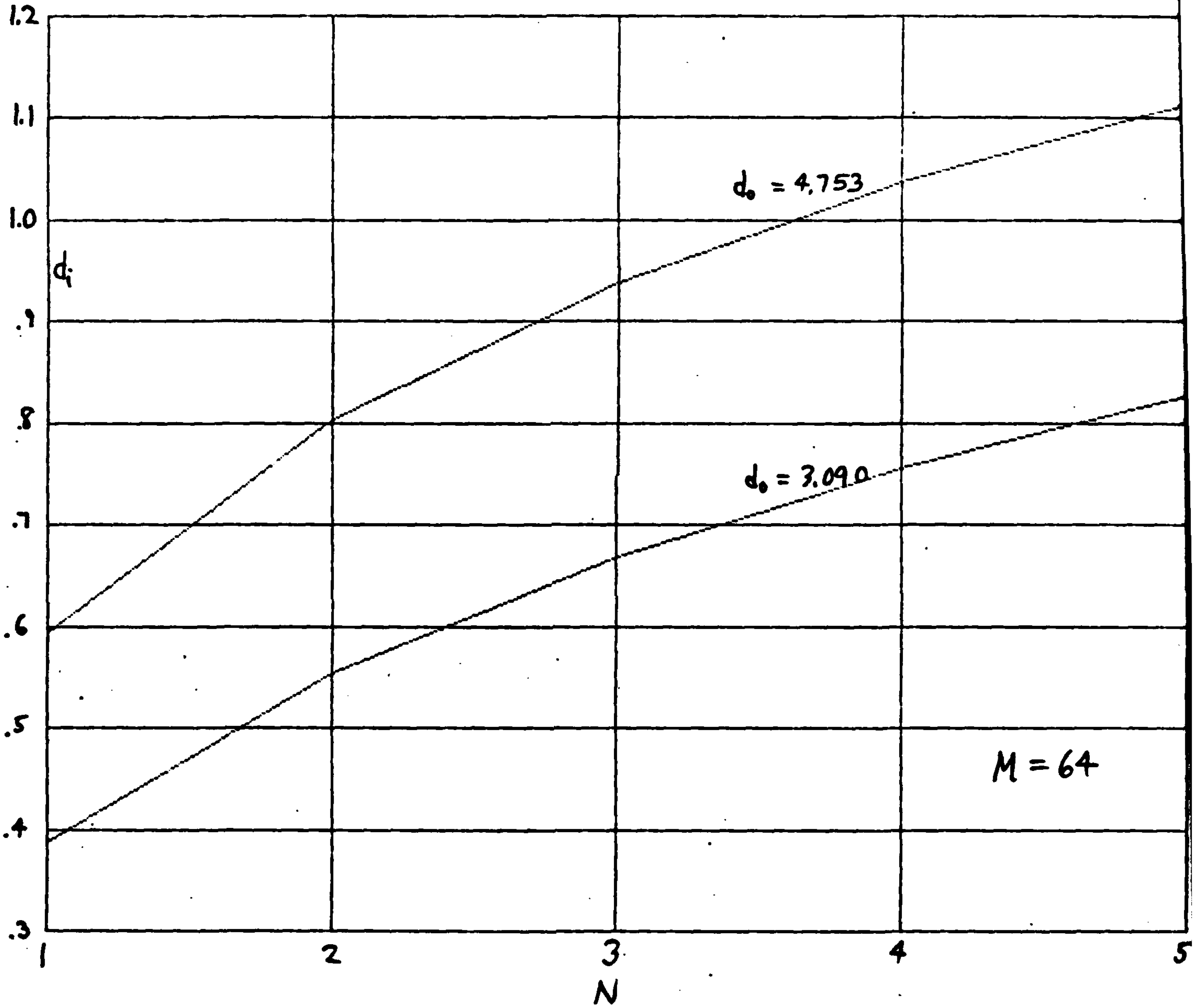

Figure 11. Input Deflection Requirements for M=64, No Quantizers

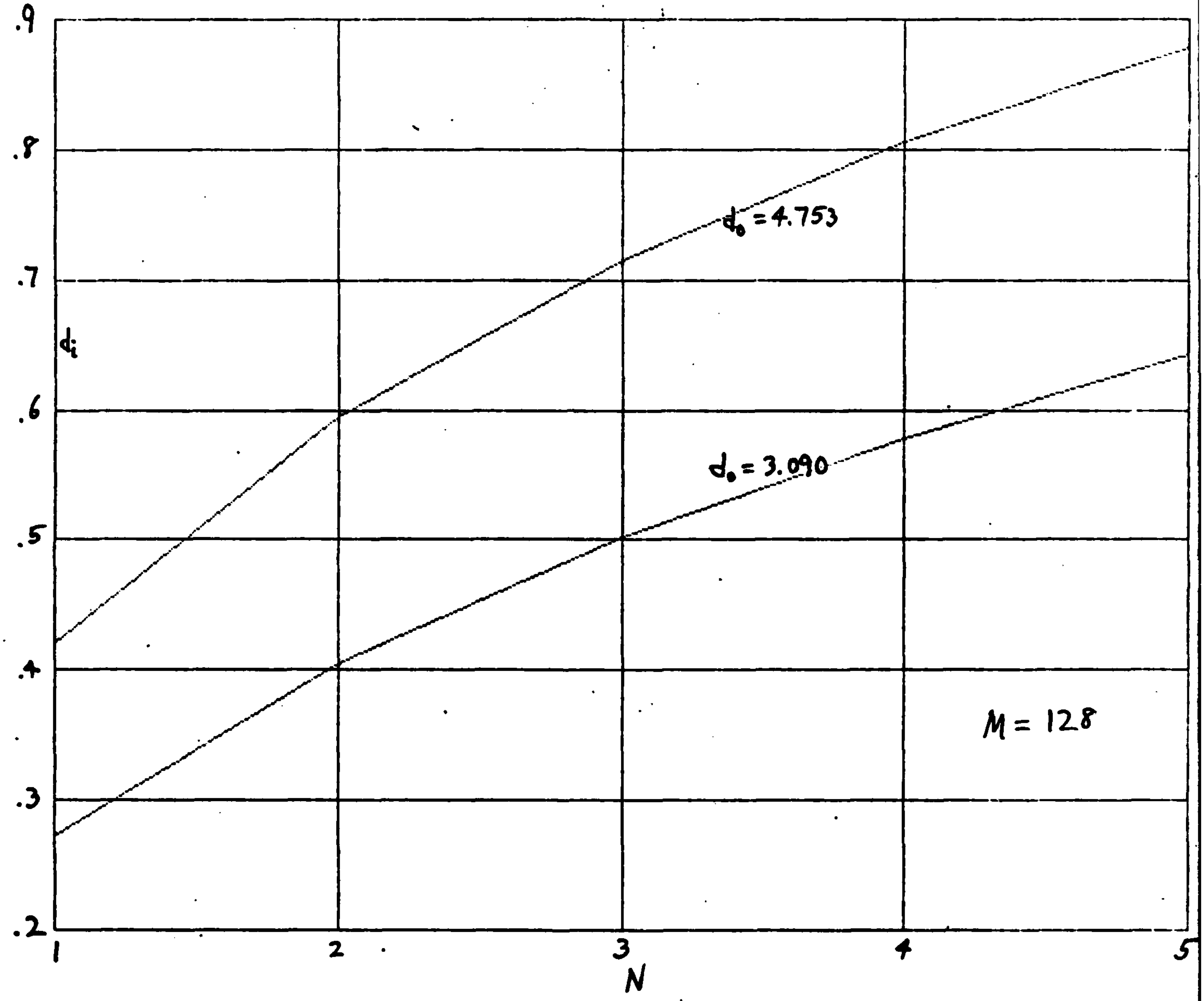

Figure 12. Input Deflection Requirements for M= 128, No Quantizers

APPENDIX A. Definition of Effective Number of Independent Inputs

Consider random variables $x_1, \ldots, x_N$ which are identically distributed (noise-only case), but not necessarily independent. Define

$$z = \sum_{n=1}^{N} x_n . \tag{A-1}$$

Then

$$\mu_z = \overline{z} = \sum_{n=1}^{N} \overline{x_n} = N\mu_x . \tag{A-2}$$

Let

$$a_n = x_n - \mu_x ; \quad \overline{a_n} = 0, \quad \overline{a_n^2} = \overline{(x_n - \mu_x)^2} = \sigma_x^2 , \tag{A-3}$$

and

$$\overline{a_m a_n} = \sigma_x^2 \rho_{mn} \qquad (\rho_{nn} = 1, \quad 1 \le n \le N). \tag{A-4}$$

Then

$$z - \overline{z} = \sum_{n=1}^{N} (x_n - \overline{x_n}) = \sum_{n=1}^{N} a_n , \tag{A-5}$$

and

$$\sigma_z^2 = \overline{(z - \overline{z})^2} = \sum_{m,n=1}^{N} \overline{a_m a_n} = \sigma_x^2 \sum_{m,n=1}^{N} \rho_{mn} . \tag{A-6}$$

Define

$$N_{eff} = \frac{\mu_z^2 / \sigma_z^2}{\mu_x^2 / \sigma_x^2} = \frac{N^2}{\sum_{m,n=1}^{N} \rho_{mn}} . \tag{A-7}$$

$$\left.\begin{array}{l} \text{If } \rho_{mn} = \delta_{mn} , \ N_{eff} = N. \\ \text{If } \rho_{mn} = 1 , \ N_{eff} = 1. \end{array}\right\} \tag{A-8}$$

These check intuition.

So the results in this memorandum could be used for dependent inputs $x_1, \ldots, x_N$, if N is replaced by

$$N_{eff} = \frac{N^2}{\sum_{m,n=1}^{N} \rho_{mn}} . \tag{A-9}$$

This applies even if N_{eff} is not an integer; this yields no methematical difficulty.

APPENDIX B. Program for System with Quantizers

The following Basic program is written for the Hewlett Packard 9845A. The quantizer parameters are entered in lines 40, 50, and 60. d_i, N, and M are input in lines 10, 20, and 30 respectively.

```
10    INPUT Di               !  Di>=0    INPUT DEFLECTION
20    N=5                    !  N>=1     NUMBER OF INPUT CHANNELS
30    M=32                   !  M> 0     NUMBER OF TERMS IN ACCUMULATOR
40    L=7                    !  L>=1     NUMBER OF LEVELS -1 IN QUANTIZER
50    DATA 0,1,2,3,4,5,6,7
51      REM    L EQUI-SIZED VERTICAL JUMPS FROM 0 UP TO L
60    DATA 0,.33333,.66667,1,1.33333,1.66667,2
61      REM    L-1 EQUI-SPACED HORIZONTAL JUMPS FROM MEAN TO 2*SIGMA
70    DIM H(0:7),V(1:7)
80    MAT READ H,V
90    Muy0=Muy1=H(L)
100   Y02av=H(L)^2
110   T1=N-1
120   FOR L1=1 TO L
130   T2=FNP(V(L1))
140   T3=T2^T1
150   Q0=T3*T2
160   Q1=T3*FNP(V(L1)-Di)
170   T4=H(L1)-H(L1-1)
180   Muy0=Muy0-Q0*T4
190   Muy1=Muy1-Q1*T4
200   Y02av=Y02av-Q0*(H(L1)^2-H(L1-1)^2)
210   NEXT L1
220   Sigy0=SQR(Y02av-Muy0^2)
230   Do=SQR(M)*(Muy1-Muy0)/Sigy0          !  OUTPUT DEFLECTION
240   PRINT "L =";L,"M =";M,"N =";N
250   PRINT "Di =";Di,"Do =";Do
260   STOP
270   DEF FNP(P0)      ! PHI
280   P=ABS(P0)
290   IF P<7 THEN 320
300   P=0
310   GOTO 350
320   P=1/(1+.2316419*P)
330   P=P*(.31938153-P*(.356563782-P*(1.781477937-P*(1.821255978-P*1.330274430))))
340   P=P*EXP(-.5*P0^2)*.398942280401
350   IF P0<0 THEN 370
360   P=1-P
370   RETURN P
380   FNEND
390   END
```

APPENDIX C. Program for System without Quantizers

The following Basic program is written for the Hewlett Packard 9845A; it is essentially the same as Ref. 1, appendix B. The inputs are r and N in lines 30 and 40. Observation of the comments in lines 170-190 is

```
10   REM      TM No. TC-13-75, APPENDIX B
20   REM      NOTICE LINES  A, B, C, AND D BELOW
30   DEF FNF(R)
40 A:       N=8                                 !  INPUT N >= 2
50   C1=1/SQR(2*PI)
60   S1=-8
70   S2=8
80   S3=(FNS(S1,R,N,C1)+FNS(S2,R,N,C1))*.5
90   S4=0
100  S5=2
110  S6=(S2-S1)*.5
120  S7=2/3
130 Loop: FOR S8=1 TO S5-1 STEP 2
140  S4=S4+FNS(S1+S6*S8,R,N,C1)
150  NEXT S8
160  Fold=F
170 B:      F=(S3+2*S4)*S6*S7-.01541            ! RHS OF (35) SUBTRACTED
180 C:      IF S5<128 THEN 200                  ! NEED MORE THEN 128 ?
190 D:      IF ABS(F-Fold)<1E-5 THEN RETURN F   ! ERROR TOLERANCE OK ?
200  S3=S3+S4
210  S4=0
220  S5=S5*2
230  S6=S6*.5
240  GOTO Loop
250  FNEND
260  DEF FNS(X,R,N,C1)
270  T1=FNP(X,C1)
280  T2=EXP(-.5*X^2)
290  T3=X-R
300  IF N=2 THEN 330
310  T4=T1^(N-2)
320  GOTO 340
330  T4=1
340  S=C1*X*T4*((N-1)*T2*FNP(T3,C1)+T1*EXP(-.5*T3^2)-N*T2*T1)
350  RETURN S
360  FNEND
370  DEF FNP(P0,C1)     ! PHI
380  P=ABS(P0)
390  IF P<7 THEN 420
400  P=0
410  GOTO 450
420  P=1/(1+.2316419*P)
430  P=P*(.31938153-P*(.356563782-P*(1.781477937-P*(1.821255978-P*1.33027443)
440  P=C1*P*EXP(-.5*P0^2)
450  IF P0<0 THEN 470
460  P=1-P
470  RETURN P
480  FNEND
```

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1. A. H. Nuttall, "Signal-to-Noise Ratio Requirements for Greatest-of Device followed by Integrator", NUSC Technical Memorandum TC-13-75, 24 July 1975

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